Proof. — We have $q_1\delta_1 = p\delta_0$ by the corollary to Proposition 5. Therefore, it is sufficient to prove (2) for $k \geq 2$. Set $\sigma = i_k$, and let us consider the surface M_σ obtained by the $(\sigma - 1)-$th blowing up in the process to get M from M_1. We may say that M_σ is the surface obtained by the blowing down of L_{h+1}, L_h, ..., L_{k+1} successively from M. Let $\pi_\sigma : M \to M_\sigma$ be the contraction mapping. As in the previous sections, let us denote the proper images of $\overline{C}$, $\overline{C}_k$, E_i in M_σ by $\overline{C}^{(\sigma)}$, $\overline{C}_k^{(\sigma)}$, $E_i^{(\sigma)}$ respectively. By Theorem 3, $\overline{C}_{k+1}^{(\sigma)}$ intersects transversely $E_\sigma^{(\sigma)}$ at the same point $Q = \pi_\sigma(L_{k+1} \cup \cdots \cup L_{h+1})$ *as* $\overline{c}^{(\sigma)}$. Hence, the functions f and g_{k+1} on $M_\sigma$have the same indetermination point $Q \in E_\sigma^{(\sigma)}$. Let

$$P_f^{(\sigma)} = \sum_{i=0}^{\sigma} \nu_i E_i^{(\sigma)},\ P_{gk+1}^{(\sigma)} = \sum_{i=0}^{\sigma} \overline{\nu}_i E_i^{(\sigma)}$$

be the pole divisor of f and g_{k+1} on M_σ respectively. Let $\overline{\delta}_0, \overline{\delta}_1, \cdots, \overline{\delta}_k$ be the order of the pole of g_{k+1} on E_{j_0} $(= E_0)$, E_{j_1} $(= E_1)$, $\cdots$, E_{j_k}. We have $\overline{\delta}_0 = \overline{\nu}_{jo}, \overline{\delta}_1 = \overline{\nu}_{j_1}, \cdots, \overline{\delta}_k = \overline{\nu}_{j_k}$. The coefficients $\nu_i, \overline{\nu}_i (i = 0, 1, \cdots, \sigma)$ are the solutions of the following equations:

$$\sum_{j=0}^{\sigma} (E_i^{(\sigma)} \cdot E_j^{(\sigma)}) \nu_j = \begin{cases} 0 (i \neq \sigma) \\ d_{k+1} (i = \sigma), \end{cases}$$

$$\sum_{j=0}^{\sigma} (E_i^{(\sigma)} \cdot E_j^{(\sigma)}) \overline{\nu}_j = \begin{cases} 0 (i \neq \sigma) \\ 1 (i = \sigma). \end{cases}$$

Hence, by Lemma 4, we have $\nu_i = d_{k+1}\overline{\nu}_i$ for all $i = 0, 1, \cdots, \sigma$. In particular,

$$\delta_i = \overline{\delta}_i \cdot d_{k+1},\ (i = 0, 1, \cdots, k).$$

Therefore, in order to prove (2), it is sufficient to prove

$$(3)\ q_k \overline{\delta}_k \in \mathbb{N}\overline{\delta}_0 + \mathbb{N}\overline{\delta}_1 + \cdots + \mathbb{N}\overline{\delta}_{k-1}.$$

By Theorem 3, $\overline{C}_k^{(\sigma)}$ intersects $E_{j_k}^{(\sigma)}$ transversely and does not inter- sects other components $E_i^{(\sigma)} (i \neq j_k)$. We have

$$\overline{\delta}_k = (P_{g_{k+1}}^{(\sigma)} \cdot \overline{C}_k^{(\sigma)})$$

$$= (\overline{C}_{k+1}^{(\sigma)} \cdot \overline{C}_k^{(\sigma)})$$

$$= (\overline{C}_{k+1}^{(\sigma)} \cdot P_{g_k}^{(\sigma)}).$$